# Perspectives of Mathematics on the Borderline of School and University

## Project title:

## Mathematics - A Key to Understanding our World

## Summary

The aim of the project is to provide an adequate picture of mathematics as a whole which illustrates the *impact of mathematical theories, methods, and applications* on the cultural history of mankind, the description of the present world, and the prediction of future developments. We are convinced that mathematics, by means of its extraordinary scientific power and intrinsic beauty, may inspire young people to *independent research* also beyond the standard topics discussed at school. This emphasizes the need for a *closer cooperation* in general, and a more regular exchange of *ideas and methods*, in particular, between universities and schools.

In the framework of this project we plan *activities of three different types* (see below) for the next three years. On the one hand, a part of these activities has already started in the past and is supposed to be continued. On the other, some new parts should be initiated this year; this refers, for example, to the several-days activities described in Subsection 3.2.2 below. We point out that the project is supported by *all* chairs of the Departments of Pure and Applied Mathematics at the University of Würzburg.

The just mentioned three kinds of activities may be distinguished as follows (for a more detailed description see Sections 3.2 and 3.3.

- **One-day activities.** These comprise *visits of professors* of the Department of Mathematics *at schools*, a periodically organized *Math Day* at the University of Würzburg, and some special activities in the framework of an (already existing) *cooperation between the Department of Mathematics and the local industry*.
- **Several-day activities.** We plan a three-day event called *JUMAX 2000*, as well as *mathematics circles* for particularly gifted students and *Combined Math Days* involving students, school teachers, professors and assistants of the Department of Mathematics, and advanced students of mathematics and computer sciences at the University of Würzburg.
- **Supporting activities.** Basically, these activities will be concerned with *teacher training seminars*, *advertising*, and *public relations*.

## 1. Prerequisites and facilities at the University of Würzburg

Several members of the Departments of Pure and Applied Mathematics of the University of Würzburg have some experience in organizing activities of the type considered in this application. We restrict ourselves to mentioning a few names without aiming for a complete coverage.

Prof. Appell has organized a *two-weeks workshop* under the auspices of the association "Bildung und Begabung" (instruction and talent). During that workshop a group of 13th grade school students acquired some basic knowledge on differential equations and stability theory, including applications to mathematical aspects of the Hodgkin-Huxley neural theory of nerve conduction.

Prof. Dobrowolski has organized the very successful *Math Day 1999* at the University of Würzburg. The various lectures during the Math Day 1999 have been attended by about 50 students from 10 different schools.

Prof. Klingenberg has participated in the well-established program *Mathematical talks at school* (see Section 2 below). He has given lectures in different fields to a large audience of up to 100 students.

Prof. Weigand is responsible for the educational parts (didactics) of the teachers' education at the University of Würzburg. He supervises the field experience of the students and has good relations to many schools in the surroundings of Würzburg. He has organized a lot of teacher training programs, math days and visits of school classes at the Department of Mathematics (formerly of the University of Giessen, now at the University of Würzburg).

We point out that many other colleagues from the Faculty of Mathematics and Computer Sciences have shown devotion to, and ability in, attracting young people to their research interest. We are convinced that only researchers with a great scientific enthusiasm and optimism may inspire school students to chose mathematics as "life study".

Finally, it is almost self-evident that there is an excellent computer and workstation equipment at our faculty which guarantees to carry out successfully many problems arising in the computer-aided mathematical modelling of applied sciences.

## 2. Preparatory work connected to the project

Some of the activities described in the Summary already belong to a well-established cooperation between our department and some schools in Würzburg. This refers, for example, to *mutual visits* of both the staff of the Department of Mathematics and students of various schools in the Würzburg area.

Members of our faculty have repeatedly visited high schools (Gymnasien) in the wider area surrounding Würzburg and given lectures in front of 11th to 13th grade students. Examples of titles of such lectures are *Mathematics helps us understand star formation* or *The role of computers in applying mathematics to physical problems*. These lectures were typically given during the school session, where teachers gladly offered their classtime usually alloted to mathematics and physics lessons. There were attended by up to 100 students. From the questions asked by the students afterwards it became clear that the students

lacked any notion of what is actually being done in contemporary mathematics, and what role is played by mathematics in science and society in general. A genuine interest was generated among the students and requests for a repeat of such lectures. This motivated us to come up with a larger program as presented in this application.

Another "traditional" topic which is worth mentioning is the *Math Day* organized the last time in February 1999 at the Department of Mathematics of the University of Würzburg. The program included some relevant mathematical topics of current interest, ranging from advanced theoretical fields like *Chaotic dynamical systems* to more application-oriented subjects like *Visualization of liquid flows*.

## 3. Presentation of the project

In what follows, we describe the objectives of the project as a whole, illustrate the methods for their realization, and trace some organizational aspects.

**3.1. Aims and scope.** The main goal of our project is very simple: to increase the number of students to enrol in our faculty. We are convinced that this goal can only be achieved if one takes into account some evident facts and emphasizes the resulting consequences:

- High school students have to have a belief of mathematical sciences which adequately reflects their *meaning and importance*, both in the present world and in the cultural history of mankind. We see this a good basis for studying mathematics.
- Teachers have to be motivated to *stay in contact with the university*. This not only provides teachers with *stimulation and new ideas* for their own work, but it also gives the university the opportunity to influence the situation at school. In particular, this would allow us to become aware of gifted students in mathematics.
- *Public relations* are of vital importance. A decline of interest of the general public in mathematical sciences may in fact increase the risk of "truncated" mathematical departments whose only task is the fulfilment of short-term industrial commissions. The successful work of any Department of Mathematics should therefore be constantly demonstrated to the general public, also because in the long run many departments will be looking for private funds.
- There is a deplorable phenomenon which is sometimes vaguely described as "math anxiety". We think that *all* students should be inspired for the beauty of mathematics. Afterwards, the gifted students have to be motivated for a study of mathematics.
- A group of particularly talented students should be given, within a several-days activity, an idea of how *mathematical research* actually looks like. To this end, we have created the *Math Days* to solve problems in pure mathematics, to illustrate the mathematical modelling of applied problems, and to teach the use of computers as an important tool in the contemporary mathematical and natural sciences.

**3.2. Methods.** In order to achieve the above-mentioned objectives we plan to organize seven partial activities with a long-term perspective which are described in the following

subsections. These activities fall naturally into one of the three categories *one-day activities*, *several-days acitivities*, or *supporting activities*.

- One-day activities comprise *visits of professors* of the Department of Mathematics *at schools* which are already well-established (see Section 2). These visits should include not only a mathematical talk or lecture which is accessible to a non-specialized audience, but also informal discussions with students. All 45 high schools (Gymnasien) of the governmental district of Lower Franconia will be involved in this activity.

  Another activity of similar type is a *Math Day* at the University of Würzburg, containing lectures, workshops and competitions. Such Math Days have been repeatedly organized by the University of Würzburg for many years. Usually this day is a weekday, and the students in their last year of high school get the opportunity to visit all faculties of the university. Besides the staff of the Department of Mathematics, also mathematics students are involved in the organization of this day. In the future it should be discussed whether the University of Würzburg will take part in a Math Day as it is organized in Baden-Württemberg and Hessen.

  A further activity aims at a possible *cooperation between the University of Würzburg and the local industry* which is motivated by a deplorable lack of interconnections between today's mathematical education of future teachers, on the one hand, and their experience in working on "real-life problems", on the other.

- Several-days activities are also planned. For instance, the *JUMAX 2000* is a three-day event which will take place on July 6 - 8, 2000. Detailed information on the program can be found on the web site `http://www-uni-wuerzburg.de/presse/mitteilungen/p00-041w.html`. Even if this event addresses itself not exclusively to pupils or students, it is nevertheless important for us, since we are looking for contacts to the local industry and enterprises. As a matter of fact, we are interested in modelling "real-life" problems during our Math Day, and the local industry might serve as an interesting source for such problems. Moreover, we want local enterprises not just for funding, but as partners with a real interest in our activities.

  The idea of a *Math Circle* consists in providing gifted students with the opportunity for regular meetings at the university. The organization of this groups is done by Prof. Dobrowolski from the Department of Applied Mathematics. The meetings will take place in a three or four weeks rhythm, one group comprises about 10 or 15 students. We are also interested in students of grade 7 - 10, because we are convinced that the promotion of mathematical talents may and should start as early as possible.

  Apart from school students and teachers, also students from the University of Würzburg should be integrated into these meetings. It is one goal to encourage the students to participate in *mathematics competitions* like "Thinkquest" or "Bundeswettbewerb Mathematik". Finally, we think about having a group of students (and teachers and teacher students) who will be able and ready to answer questions

from other students via Internet, i.e. some kind of German version of the American "Ask Dr. Math" (see `http://forum.swarthmore.edu/`).

The problem of finding a suitable "jour fixe" for these meetings is still open, but probably Saturday may be a good choice.

Some kind of extended version of the Math Day is the *Combined Math Days* program described in Subsection 3.3. We will give interested and talented students the opportunity to come to the university and work on three consecutive days in small groups on mathematical problems. Experts will assist the students, and problems of both pure and applied mathematics should be studied. We believe that groups of about 5 - 7 students who are assisted by at least one member of the staff of the Department of Mathematics, one or two high school teachers, and some teacher students from the university will be ideal. It should also be considered whether or not it makes sense to assign credit points to the teacher students for participating in this activity.

The preparation of these projects is supposed to be done in cooperation with teachers from schools in and around Würzburg, as well as teacher students of our Department. Obviously, we will take into account the experience of formerly successful projects, as for instance the "modelling weeks" of the University of Kaiserslautern or the TUMSS project at the Technical University of Munich.

- Supporting activities are as follows. For a long-term consideration of our project we depend on a good cooperation with school teachers and students and with the local industry. In view of our main goals described above, we consider the following initiatives quite important, although they seem to have "only" supporting character for the other activities.

  The chair for Didactics of Mathematics at the University of Würzburg is regularly organizing *Teacher Advanced Training Seminars*. The next meeting of this type will take place on July 12, 2000. These meetings give us the possibility of (re-)establishing an intensive contact to school teachers and presenting some activities of our Department. In this connection we also ask the teachers to help us in selecting appropriate students for the Math Day or Combined Math Days.

  One of the best ways to appreciate the work of the participating students is an article on their activities in the local newspaper. For this reason, *public relation* is quite important to us. As a matter of fact, in this way our work becomes not only known to a wider general public, but as a "by-product" it may also open the doors for obtaining financial support from the local industry.

**3.3. Organization and realization.** The various activities described above lie in the responsability of the applicant and some of his colleagues. More precisely, the following organizational aspects should be mentioned.

- **School visits by members of the Department.** All 45 high schools (Gymnasien) of the district Lower Franconia have received a preliminary invitation. The visits will

be organized upon request of each school by the chair for Didactics of Mathematics of the University of Würzburg.

- **Math Day.** The organization will also be in the responsability of the chair for Didactics of Mathematics of the University of Würzburg.

- **JUMAX 2000.** Prof. Golitschek (Department of Mathematics) and Prof. Wagner (Department of Computer Sciences) are organizing the contributions of our faculty to JUMAX 2000. All chairs of the Departments of Pure and Applied Mathematics are involved in this activity. The complete program can be found on the web site `http://www.JUMAX2000.de`.

- **Math Circle.** This event will be organized by Prof. Dobrowolski (Department of Applied Mathematics).

- **Combined Math Days**, constituting the most ambitious part of the project, require a particularly careful and prospective organization. To this end, a special committee has been convened to carry out the preparation and realization of the Combined Math Days in this and the following years. The committee includes 6 members of both departments (Prof. Appell, Prof. Dobrowolski, Dr. Greiner, Dipl.-Math. Grahl, Prof. Klingenberg, Prof. Weigand). The chair of Didactics of Mathematics is responsible for establishing the contact to the teachers. The selection of the topics to be dealt with during the Combined Math Days will be done by the committee in cooperation with the teachers and teacher students involved. The results obtained in the framework of this activity will be published on the Internet and/or in a special brochure which may also serve as an orientation for forthcoming activities.

  The *accomodation* of the participants of Math Days will be at a conference centre in or near Würzburg. There are several possibilities in or around Würzburg ("Frankenwarte", "Burg Rothenfels", Benediktushöhe Retzbach" etc.). Of course, there will be also a cultural program during the meeting.

  The *single steps* in the organization of the Combined Math Days read as follows:

  - Meeting of the members of the university to discuss possible topics for the project
  - Meeting of the members of the university with high school teachers and teacher students to split the group into subgroups
  - Preparation of the different topics in subgroups; this may require several subsequent meetings

  A possible *timetable* for the Combined Math Days may be as follows:

  - Tuesday afternoon: registration; introduction to the subject; organization of subgroups
  - Wednesday: working on the project

- Wednesday evening: discussions on general mathematical problems, possibily involving some invited guest speaker
- Thursday: working on the project
- Thursday evening: cultural event (theater, cinema, etc.)
- Friday morning: discussion of the results within the subgroup; preparing the presentation of results
- Friday noon: presentation of the results to the plenum
- Friday afternoon: closure of the project

To obtain the permission for the teachers to attend this activity we have contacted the supervisor of the local school board. We have been assured that the project will be recognized as an advanced training seminar for teachers. The preliminary meetings for organizing the project will be in the free time of the teachers and thus will not affect their remaining duties.

- **Teacher Advanced Training Seminars** will be organized by the chair for Didactics of Mathematics in cooperation with the supervisor of the local school board and the corresponding teachers' organizations (Philologenverband, BLLV). For this activity we already dispose of some minor funding from the local industry.
- **Public relations** are ensured by good contacts to both the public relation centre of the University of Würzburg and the local press. So the organization and transfer of information will run without problems.

**3.4. General time schedule.** The following time schedule illustrates the "state-of-affairs" and the further concerted action.

- **School visits.** We have offered the schools some appointments in June/July 2000, as well as between September and December 2000.
- **Math Days.** The next Math Day is scheduled for February 2001. We will start our preparations in October 2000.
- **JUMAX 2000.** The preparations for JUMAX 2000 are finished. The event will take place on July 6 - 8, 2000.
- **Math Circle.** Our concept will be presented to the teachers during the Teacher Advanced Training Seminar on July 12, 2000. In September all high schools will receive detailed written information. We also assume that we will receive a positive resonsance as a consequence of our visits at the schools. The Math Circle will start in October 2000, organized by Prof. Dobrowolski.
- **Combined Math Days.** This activity is scheduled for April or May, 2001; this will give us sufficient preparation time during the fall term. To this end, we agreed on the following preliminary timetable:

- September: Informing the teachers and schools; establishing contacts to teachers who are interested in joining the project
- October: Internal meetings of the university group
- November: Meetings of the extended project groups with teachers and teacher students
- December/January/February: Organization of the projects, reflecting the methodology, collecting information, contacting the local industry, discussing possible problems
- January/February: Selection of the students for the Combined Math Days
- March/April: Preparation of the program in detail

- **Advanced Teacher Training.** The first event (high school teachers, July 2000) is organized by Prof. Weigand, the second event (elementary school teachers, October 2000) by Dr. K. Appell, and the third event (vocational school teachers, February 2001) by Dr. Glaser, all belonging to the scientific staff of the chair for Didactics of Mathematics. Prof. Weigand will contact the local press.

**3.5. Further details.** In this final section we describe the criteria for selecting the participants and evaluating the project.

- **Criteria for selecting the participants.** The following four different groups are involved in our activities:
  - Members of the Departments of Pure and Applied Mathematics
  - High school teachers
  - Upper stage students of mathematics or computer sciences
  - Teacher students

  The high school teachers are very important and will play a crucial role in our concept, since they will support the selection of the students for the Math Circle and the Combined Math Days. By means of a further competition during the Math Days we may then select particularly talented students.

  As already mentioned, teachers will be included in our project. We think that this will not only favour the interconnections between topics taught at school and those covered by our project, but will also motivate the teachers and increase their engagement in selecting suitable students and realizing the projects.

- **Advertising and public relations.** Our activities will be advertised in several ways:
  - Flyers sent by the local school board to all high schools of Lower Franconia
  - Publication on the Internet
  - Publication in local newspapers

- Personal contacts to teachers during the teacher training
- Personal contacts to the teachers' professional council ("Philologenverband")

- **Criteria for evaluating the project.** To ensure an efficient internal and external evaluation of all activities there will be three different levels of evaluation.

  - **Questionnaire.** After our visits at school the teachers whose classes have been involved in the meeting are requested to provide us with a feedback. This may be done by means of a questionnaire filled out the day after the visit ("What is your opinion about the presentation?" "Did the presentation improve your knowledge about mathematics or about studying mathematics?" "Which point did you miss?" etc.).

    A questionnaire is also to be filled out after the Math Circle by each group involved.

  - **Interviews.** During the Math Day, the JUMAX 2000, and the Combined Math Days some visitors will be interviewed. To this end, a special questionnaire has to be outlined. The interviews may be done by student teachers.

  - **Expert evaluation.** During the Combined Math days either a teacher or some teacher students will be concerned with evaluation. They are supposed to design a suitable questionnaire which has to be filled out by all participants. Moreover, they may also interview some of the participating students. We are interested in how the students' mathematical belief has changed in the course of the project, and also if their intention to study mathematics has been positively influenced.